NAG Toolbox for MATLAB

g01jd

1 Purpose

g01jd calculates the lower tail probability for a linear combination of (central) χ^2 variables.

2 Syntax

```
[prob, ifail] = g01jd(method, rlam, d, c, 'n', n)
```

3 Description

Let $u_1, u_2, \ldots, u_n$ be independent Normal variables with mean zero and unit variance, so that $u_1^2, u_2^2, \ldots, u_n^2$ have independent χ^2-distributions with unit degrees of freedom. g01jd evaluates the probability that

$$\lambda_1 u_1^2 + \lambda_2 u_2^2 + \cdots + \lambda_n u_n^2 < d\left(u_1^2 + u_2^2 + \cdots + u_n^2\right) + c.$$

If $c = 0.0$ this is equivalent to the probability that

$$\frac{\lambda_1 u_1^2 + \lambda_2 u_2^2 + \cdots + \lambda_n u_n^2}{u_1^2 + u_2^2 + \cdots + u_n^2} < d.$$

Alternatively let

$$\lambda_i^* = \lambda_i - d, \qquad \text{for } i = 1, 2, \ldots, n,$$

then g01jd returns the probability that

$$\lambda_1^* u_1^2 + \lambda_2^* u_2^2 + \cdots + \lambda_n^* u_n^2 < c.$$

Two methods are available. One due to Pan 1964 (see Farebrother 1980) makes use of series approximations. The other method due to Imhof 1961 reduces the problem to a one-dimensional integral. If $n \geq 6$ then a non-adaptive method described in d01bd is used to compute the value of the integral otherwise d01aj is used.

Pan's procedure can only be used if the λ_i^* are sufficiently distinct; g01jd requires the λ_i^* to be at least 1% distinct; see Section 8. If the λ_i^* are at least 1% distinct and $n \leq 60$, then Pan's procedure is recommended; otherwise Imhof's procedure is recommended.

4 References

Farebrother R W 1980 Algorithm AS 153. Pan's procedure for the tail probabilities of the Durbin–Watson statistic *Appl. Statist.* **29** 224–227

Imhof J P 1961 Computing the distribution of quadratic forms in Normal variables *Biometrika* **48** 419–426

Pan Jie–Jian 1964 Distributions of the noncircular serial correlation coefficients *Shuxue Jinzhan* **7** 328–337

5 Parameters

5.1 Compulsory Input Parameters

1: **method – string**

Indicates whether Pan's, Imhof's or an appropriately selected procedure is to be used.

method = 'P'

Pan's method is used.

method = 'I'

Imhof's method is used.

method = 'D'

Pan's method is used if λ_i^*, for $i = 1, 2, \ldots, n$ are at least 1% distinct and $n \leq 60$; otherwise Imhof's method is used.

Constraint: **method** = 'P', 'I' or 'D'.

2: **rlam**(**n**) – **double array**

The weights, λ_i, for $i = 1, 2, \ldots, n$, of the central χ^2 variables.

Constraint: **rlam**$(i) \neq$ **d** for at least one i. If **method** = 'P', then the λ_i^*, for $i = 1, 2, \ldots, n$, must be at least 1% distinct; see Section 8.

3: **d** – **double scalar**

d, the multiplier of the central χ^2 variables.

Constraint: **d** ≥ 0.0.

4: **c** – **double scalar**

c, the value of the constant.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The dimension of the array **rlam**.

n, the number of independent standard Normal variates, (central χ^2 variates).

Constraint: **n** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

work

5.4 Output Parameters

1: **prob** – **double scalar**

The lower tail probability for the linear combination of central χ^2 variables.

2: **ifail** – **int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **n** < 1,
or **d** < 0.0,
or **method** $\neq$ 'P', 'I' or 'D'.

ifail $= 2$

On entry, **rlam**$(i) =$ **d** for all values of i, for $i = 1, 2, \ldots, n$.

ifail $= 3$

On entry, **method** $=$ 'P' yet two successive values of the ordered λ_i^*, for $i = 1, 2, \ldots, n$, were not at least 1% distinct.

7 Accuracy

On successful exit at least four decimal places of accuracy should be achieved.

8 Further Comments

Pan's procedure can only work if the λ_i^* are sufficiently distinct. g01jd uses the check $|w_j - w_{j-1}| \geq 0.01 \times \max(|w_j|, |w_{j-1}|)$, where the w_j are the ordered nonzero values of λ_i^*.

For the situation when all the λ_i are positive g01jc may be used. If the probabilities required are for the Durbin–Watson test, then the bounds for the probabilities are given by g01ep.

9 Example

```
method = 'P';
rlam = [-9;
     -7;
     -5;
     -3;
     -1;
     2;
     4;
     6;
     8;
     10];
d = 1;
c = 0;
[prob, ifail] = g01jd(method, rlam, d, c)
```

```
prob =
    0.5749
ifail =
           0
```